# Exercise 55

For the following exercises, sketch a graph of the quadratic function and give the vertex, axis of symmetry, and intercepts.

$$f(x) = x^2 - 5x - 6$$

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**Solution**

In order to more easily graph the quadratic function, write it in vertex form by completing the square. The following algebraic identity is necessary.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -5$, which means $B = -\frac{5}{2}$ and $B^2 = \frac{25}{4}$. Add and subtract $\frac{25}{4}$ from the right side and use the identity.

$$\begin{aligned} f(x) &= \left(x^2 - 5x + \frac{25}{4}\right) - 6 - \frac{25}{4} \\ &= \left(x + \left(-\frac{5}{2}\right)\right)^2 - \frac{49}{4} \\ &= \left(x - \frac{5}{2}\right)^2 - \frac{49}{4} \end{aligned}$$

Therefore, the vertex is $\left(\frac{5}{2}, -\frac{49}{4}\right)$, and the axis of symmetry is $x = \frac{5}{2}$. To determine the $y$-intercept, set $x = 0$.

$$f(0) = \left(0 - \frac{5}{2}\right)^2 - \frac{49}{4} = \left(\frac{25}{4}\right) - \frac{49}{4} = -\frac{24}{4} = -6$$

Therefore, the $y$-intercept is $(0, -6)$. To get the $x$-intercept, set $y = 0$ and solve the equation for $x$.

$$\begin{aligned} 0 &= \left(x - \frac{5}{2}\right)^2 - \frac{49}{4} \\ \frac{49}{4} &= \left(x - \frac{5}{2}\right)^2 \end{aligned}$$

Take the square root of both sides.

$$\sqrt{\frac{49}{4}} = \sqrt{\left(x - \frac{5}{2}\right)^2}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x - \frac{5}{2}$.

$$\left|x - \frac{5}{2}\right| = \frac{7}{2}$$

Remove the absolute value sign by placing $\pm$ on the opposite side.

$$x - \frac{5}{2} = \pm\frac{7}{2}$$

Add 5/2 to both sides.

$$x = \frac{5}{2} \pm \frac{7}{2}$$

This means $x = \{-1, 6\}$, and the $x$-intercepts are $(-1, 0)$ and $(6, 0)$. A graph of the function is shown below.